Supporting Research For Bridges in Mathematics

The approaches to teaching and learning in *Bridges in Mathematics* are grounded in research. The following is an annotated bibliography of some of the books and articles that have most dramatically influenced the development of the program. We have included this bibliography to give you some information about the foundations of the program, as well as to direct you toward reading that you may find helpful and informative. You'll notice that some resources are a bit older than others. We began developing this program years ago, when these resources were current. We feel that their value has withstood the test of time, but we have also included more current resources that have informed the program as well.

Bennett, Albert B. and L. Ted Nelson. 2004. *Mathematics for Elementary Teachers: A Conceptual Approach*, 6th ed. New York: McGraw Hill.

Bennett and Nelson's textbook for pre-service and in-service teachers of elementary mathematics provides a wealth of information about mathematical concepts. It includes activities that can be used with children, as well as activities that help teachers deepen and strengthen their own mathematical understandings. The textbook has an accompanying activity book that provides more extensive concept development through hands-on mathematics activities and problem solving.

Beto, Rachel. 2004. "Assessment and Accountability Strategies for Inquiry-Style Discussions." *Teaching Children Mathematics,* 10 (9): 450–454.

This lively article, written by a practicing classroom teacher, discusses teaching and assessment strategies that boost participation and hold all students accountable for inquiry-style discussions. The article includes many intelligent and realistic tips for promoting student discourse.

Bresser, Rusty. 2003. "Helping English-Language Learners Develop Computational Fluency." *Teaching Children Mathematics,* 9 (6): 294–299.

The author outlines ten strategies for helping English-language learners become computationally fluent. The strategies include specific questioning techniques, the use of wait time following a question, and ways for teachers to modify their speech to make it more comprehensible for English-language learners. Bresser also stresses the value of cooperative learning strategies like think-pair-share and asking some English-language learners to act as English experts for their peers.

Burns, Marilyn and Robyn Silbey. 2000. *So You Have to Teach Math? Sound Advice for K–6 Teachers,* Sausalito: Math Solutions Publications.

This slender volume provides honest and straightforward answers to 101 questions teachers ask most frequently about standards-based math instruction. The book isn't tied to a particular math program, but teachers using *Bridges* will find clear answers and down-to-earth advice about everything from managing manipulatives to handling homework. *So You Have to Teach Math?* is particularly helpful for new teachers, teachers switching grade levels, teachers using a standards-based math program for the first time, and teachers who need to be able to provide clear but cogent explanations to parents.

Carpenter, T.P., M.L. Franke, V.R. Jacobs, E. Fennema, and S. B. Empson. 1998. "A Longitudinal Study of Invention and Understanding in Children's Multidigit Addition and Subtraction." *Journal for Research in Mathematics Education*, 29 (1): 3–20.

The role of invented strategies in developing multi-digit addition and subtraction concepts and skills is the focus of this longitudinal study of 82 children in the first through third grades. Five interviews over three years focused on base-ten concepts, strategies for word problems, use of invented strategies, and flexible use of procedures. The researchers found that 90% of the children used invented strategies. "Students who used invented strategies before they learned standard algorithms demonstrated better knowledge of base-ten number concepts and were more successful in extending their knowledge to new situations than students who initially learned standard algorithms."

Carroll, W. and D. Porter. 1997. "Invented Strategies Can Develop Meaningful Mathematical Procedures." *Teaching Children Mathematics*, 3 (7): 370–374.

The authors explore the ways in which students' invented computational procedures promote understanding. The article describes students' strategies and includes illustrations of student work.

Duckworth, Eleanor. 1987. *"The Having of Wonderful Ideas" and Other Essays on Teaching and Learning*. New York: Teachers College Press.

In this collection of Duckworth's writings on Piaget and teaching, she writes about how to create situations and "occasions" in which learners of all ages can construct their own knowledge and understandings. While based heavily in theory, Duckworth's writing is accessible to a wide audience and uses many examples of teachers' and students' experiences in the classroom.

Fosnot, Catherine Twomey and Maarten Dolk. 2001. *Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction*. Portsmouth, NH: Heinemann.

This first book in the *Young Mathematicians at Work* series provides insights on how young children develop a deep understanding of number and the operations of addition and subtraction. The authors draw from the work of Dutch mathematician Hans Freudenthal's idea of "mathematizing"—the activity of structuring, modeling, and interpreting one's "lived world" mathematically. The importance of inquiry, problem solving, and the construction of big ideas in mathematics are revealed through student work samples and classroom vignettes. The authors take a new look at the development of strategies based on landmark numbers and operations to promote computational fluency.

Fosnot, Catherine Twomey and Maarten Dolk. 2001. *Young Mathematicians at Work: Constructing Multiplication and Division*. Portsmouth, NH: Heinemann.

In the second book in the *Young Mathematicians at Work* series, Fosnot and Dolk provide strategies to help teachers turn their third through fifth grade classrooms into math workshops that encourage and reflect real-world mathematics. They examine ways to engage and support children as they construct important strategies and big ideas related to multiplication and division. Classroom visits and student work samples help define modeling and provide examples of how learners construct models in context.

Fosnot, Catherine Twomey and Maarten Dolk. 2002. *Young Mathematicians at Work: Constructing Fractions, Decimals, and Percents*. Portsmouth, NH: Heinemann.

In the third book in the *Young Mathematicians at Work* series, Fosnot and Dolk focus on how children in the intermediate grades construct deep understandings of fractions, decimals, and percents.

Fuson, Karen. 2003. "Toward Computational Fluency in Multidigit Multiplication and Division." *Teaching Children Mathematics,* 9 (6): 300–305.

This article is an excerpt from a longer paper commissioned by NCTM's Research Advisory Committee to summarize the current state of educational research. This excerpt deals specifically with multi-digit multiplication and division, and provides convincing evidence that algorithms based on the area model are far more accessible to learners than the traditional algorithms generally taught to fourth and fifth graders.

Garrison, Leslie and Jill Kerper Mora. 1999. "Adapting Mathematics Instruction for English-Language Learners: The Language-Concept Connection." In *Changing the Faces of Mathematics: Perspectives on Latinos*, edited by Luis Ortiz-Franco, Norma G. Hernandez, and Yolanda De La Cruz. Reston, VA: The National Council for Teachers of Mathematics.

In this fifth chapter of a volume on Latinos and mathematics education, Garrison and Kerper Mora discuss the "vital role of language in the development of mathematical concepts." They write about the challenges and opportunities for learning that are present in four "domains." In the first domain, both language and concepts are unknown to the student. In the second, the language is known, but the concept is unknown. In the third, the language is unknown, but the concept is known. In the fourth domain, both the language and concept are known. They outline a wide variety of strategies for making input comprehensible for students in these different domains, using familiar content to reinforce language skills and making new mathematical content accessible to students who do not yet speak the language of instruction, in this case, English.

Hiebert, J., T. P. Carpenter, E. Fennema, K. C. Fuson, D. Wearne, H. Murray, A. Oliver, and P. Human. 1997. *Making Sense: Teaching and Learning Mathematics with Understanding*. Portsmouth, NH: Heinemann.

This book describes the essential features of a classroom that promotes mathematics understanding. These essential features include the nature of classroom tasks, the role of the teacher, the social culture of the classroom, the use of mathematical tools as learning supports, and the importance of equity and accessibility. In the chapter covering mathematical tools, the authors write, "First, tools of some kind are unavoidable and essential for doing mathematics. Second, students develop meaning for tools by actively using them in a variety of situations, to solve a variety of problems. Third, using tools enables students to develop deeper meaning of the mathematics that the tools are being used to examine."

Hufferd-Ackles, Kimberly, Karen Fuson, and Miriam Sherin. 2004. "Describing Levels and Components of a Math-Talk Community." *Journal for Research in Mathematics Education*, 35 (2): 81–116.

The authors of this article outline a number of concrete actions teachers can take to encourage the growth and development of a classroom math community, including:

- asking questions that focus on mathematical thinking rather than answers.
- asking students to explain their thinking while resisting the temptation to paraphrase their ideas.
- posing problems that are open-ended.
- inviting extended descriptions of strategies and soliciting more than one way to solve a problem, even a straightforward computation.
- expecting students to take on central roles in discussion and giving them the physical and psychological space to do so.
- coaching students in their participatory roles in the discourse, either as speakers or active listeners.
- actively monitoring interactions from the side or back of the room, and giving assistance when students need clarification or support in an interaction.

Isaacs, A. C. and William M. Carroll. 1999. "Strategies for Basic-Facts Instruction." *Teaching Children Mathematics*, 5 (9): 508–514.

The authors examine the advantages of a strategies-based approach to the basic facts. "The strategies approach helps students organize the facts in a meaningful network so that they are more easily remembered and accessed.… A strategies-based approach also builds students' understanding and confidence. De-emphasizing rote memorization encourages students to use their common sense in mathematics, thus supporting concept development.… The cost in instructional time is also low: delayed practice often means less practice needed later on. Children's success at learning their facts also reassures parents about their children's mathematics program."

Jarrett, Denise. 1999. *Teaching Mathematics and Science to English Language Learners*. Portland, OR: Northwest Regional Educational Laboratory.

This is one of three publications dedicated to research-based instructional strategies published as part of the Northwest Regional Educational Laboratory's *It's Just Good Teaching* series. Each publication focuses on the diverse needs of the gifted, learning disabled, and English-language learners in an inclusive classroom. Jarrett writes, "To participate meaningfully in the academic discourse and abilities that are necessary to achieve the mathematics and science standards, teachers must help [English-language learners] to develop language skills that go beyond mere social fluency." The author includes suggestions for cooperative learning strategies, teaching through inquiry and problem solving, vocabulary development, and assessment strategies that link second-language acquisition strategies with other standards-based practices.

Maier, Eugene. 2003. *Gene's Corner and Other Nooks & Crannies*. Portland, OR: The Math Learning Center.

This collection of essays by Dr.. Eugene Maier, co-founder of The Math Learning Center, chronicles 50 years of experience as a mathematician and educator. Essays focus on such topics as standardized testing, current issues in education, how children learn mathematics, and the role of technology in math education.

Maier, Eugene. 1996. "Mathematical Swindling." *Starting Points for Visual Mathematics.* Salem, OR: The Math Learning Center.

In this article, Maier explores the common phenomenon of "mathematical swindling" in which students successfully complete mathematics coursework despite having only a superficial grasp of the concepts at hand. Through observations and interviews, Maier relates stories of how students use memorized formulas and procedures to pass exams and earn high marks without ever comprehending the mathematics they are studying. He suggests that instead of focusing on mastery of textbook procedures, good mathematics instruction should encourage students to share their insights and develop mental images and models that bring meaning to symbols and procedures.

Maier, Eugene. 1985. "Mathematics and Visual Thinking." *Washington Mathematics.* September: 21–23.

Maier summarizes a variety of readings in support of mathematics curricula that provide experiences in visual thinking for all students. Visual thinking involves three mental steps: perceiving, imaging, and portraying. Incorporating these processes into mathematics curricula leads to increased understanding, enjoyment, and the meaningful use of mathematics by all students.

Marzano, Robert J. 2003. *What Works in Schools: Translating Research into Action*. Alexandria, VA: Association for Supervision and Curriculum Development.

Marzano divides the factors affecting student achievement into three categories: school-level factors, teacher-level factors, and student-level factors. In this book, he synthesizes the research and concludes that "the impact of decisions made by individual teachers is far greater than the impact of decisions made at the school level." He describes the instructional strategies that are directly correlated with increased student achievement.

Mooney, Carol Garhart. 2000. *Theories of Childhood: An Introduction to Dewey, Montessori, Erikson, Piaget & Vygotsky*. St. Paul, MN: Redleaf Press.

This slim volume provides clear, brief summaries of theories from Dewey, Montessori, Erikson, Piaget, and Vygotsky that are relevant to work with young children. Mooney also explains how an understanding of these theories can improve classroom practices for early childhood and elementary teachers.

National Council for Teachers of Mathematics. 2000. *Principles and Standards for School Mathematics*. Reston, VA: The National Council for Teachers of Mathematics.

This publication is the 2000 revision of the NCTM Principles and Standards, first published in 1989. A whole chapter is devoted to principles of school mathematics, which include equity, curriculum, teaching, learning, assessment, and technology. The standards also outline expectations for students' mathematical learning in four grade bands: Pre-K–2, grades 3–5, grades 6–8, and grades 9–12. NCTM outlines standards related to both content (e.g., geometry) and process (e.g., communication).

National Research Council. 2002. *Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

This book is based on the premise that all students can and should become proficient in mathematics. Mathematical proficiency involves five intertwined strands: understanding mathematics, computing fluently, applying concepts to solve problems, reasoning logically, and engaging with mathematics, seeing it as sensible, useful, and doable. According to the authors, "For all students to become mathematically proficient, major changes must be made in mathematics instruction, instructional materials, assessments, teacher education, and the broader educational system."

Resnick, L. B., V. L. Bill, S. B. Lesgold, and M. N. Leer. 1991. "Thinking in Arithmetic Class." In *Teaching Advanced Skills to At-Risk Students: Views from Research and Practice*, ed. B. Means, C. Chelemer, and M. S. Knapp, 27–53. San Francisco: Jossey-Bass.

This article includes a report regarding a two-year, problem-centered program in an inner-city school that stressed understanding. The program followed one first grade and one second grade. Students made substantive improvements both years on standardized test scores. They also showed highly positive results in confidence and attitude. The article presents six key principles of the program.

Schifter, D. and C. Fosnot. 1993. *Reconstructing Mathematics Education: Stories of Teachers Meeting the Challenge of Reform*. New York: Teachers College Press.

This book brings the reader into second and third grade classrooms to observe students engaged in mathematical activity. Schifter and Fosnot use these case studies to help teachers think about how students learn mathematics and consider what it takes for teachers to learn how to teach mathematics. They write, "No matter how lucidly and patiently teachers explain to their students, they cannot understand for their students. Once one accepts that the learner must herself actively explore mathematical concepts in order to build the necessary structure of understanding, it then follows that teaching mathematics must be reconceived as the provision of meaningful problems designed to encourage and facilitate the constructive process."

Senk, S. and D. Thompson. (Eds.) 2003. *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* Mahwah, NJ: Lawrence Erlbaum Associates.

This book provides an overview of many of the standards-based mathematics curricula currently being used in elementary, middle, and high school classrooms in the United States. It also attempts to answer the question of how effective these curricula are, compared to more traditional mathematics curricula, at helping students learn and understand mathematics. Numerous evaluation studies designed to answer this question are presented. In a review of the book for the *Journal for Research in Mathematics Education* (May, 2003, 34 (3): 260–265), Jinfa Cai summarized the results of these studies as follows, "Results from all of the evaluation studies reviewed appear to point in the same direction: On standardized tests measuring computational skills and procedural knowledge, students in standards-based school mathematics curricula performed at the same level as students using traditional school curricula. In addition, students in standards-based school mathematics curricula performed better than students using traditional school curricula on specifically designed tests measuring conceptual understanding and problem solving." Cai also noted a potential conflict of interest in many of the evaluation studies, "It is worth noting that many evaluation studies reviewed in this book were conducted by individuals or groups having a close association with the curriculum evaluated."

Sutton, J. and A. Krueger. (Eds.) 2002. *ED Thoughts: What We Know About Mathematics Teaching and Learning*. Aurora, CO: Mid-continent Research for Education and Learning.

According to the authors, "The purpose of this volume is to support standards-based reform of mathematics education." They provide a wealth of research and information about teaching and learning mathematics that supports a standards-based approach to mathematics instruction. The book focuses on six major topics: mathematics for all, teaching mathematics, assessment in mathematics, mathematics curriculum, instructional technology in mathematics, and learning mathematics. The book uses a question-and-answer format; the authors pose a question and then answer it from the perspectives of research and best practices. A discussion of the implications for improving classroom instruction follows each of the forty-four question-and-answer segments. The references used to answer each question are listed on the same page as the classroom implications, making them easy to find and use.

Thompson, Tony and Stephen Sproule. 2005. "Calculators for Students with Special Needs." *Teaching Children Mathematics,* 11 (7): 391–395.

Supported by current research, Thompson and Sproule argue that calculators can be used to help students with learning disabilities develop mathematical skills and understandings. They offer guidance, including a clear flowchart, to help teachers decide when and how to have students use calculators. In conclusion, they state that the calculator can be used "as a tool that can facilitate equity and inclusion and allow all students to access the full range of rich mathematics. In addition to helping students gain access to mathematical experiences that would otherwise be inaccessible to them, using a calculator can help many students with special needs increase their self-confidence, reduce their anxiety, and increase their motivation to solve mathematical problems."

Tomlinson, Carol Ann. 1999. *The Differentiated Classroom: Responding to the Needs of All Learners*. Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson draws from research on learning, education, and change for the theoretical basis of differentiated instruction. In this book, she defines a differentiated classroom and the elements of differentiation. She describes the learning environments, instructional techniques, and standards-based assessment approaches that promote learning for all children. Her premise is that most effective teachers modify some of their instruction for students some of the time. She challenges educators to use assessment data thoughtfully to modify the content, process, and products to best meet the learning profiles, readiness, and interests of students.

Thornton, C.A. and P.J. Smith. 1988. "Action Research: Strategies for Learning Subtraction Facts." *Arithmetic Teacher,* 35 (8): 8–13.

> This research is representative of many studies that have examined the effects of incorporating thinking strategies into the learning of basic facts. First grade students whose instruction emphasized strategies significantly outperformed children receiving traditional instruction. The "strategies" group had a far greater mastery of addition and subtraction facts. They also had developed mature strategies to assist them in recalling facts. The article points out that in a final interview, approximately 70% of the time the strategies groups responded at an automatic level as compared to 44% of the traditional group.

Van de Walle, John A. 2004. *Elementary and Middle School Mathematics: Teaching Developmentally*, 5th ed. New York: Pearson Education, Inc.

> This resource for elementary and middle school mathematics teachers focuses on how to address specific mathematical topics across the elementary and middle school grades. Each chapter is devoted to a particular topic or strand, and Van de Walle then chronicles how students' understandings of that topic or strand develop as they progress through elementary and middle school. He also suggests activities for use with students at various grade levels.

Webb, Norman L. 1999. *Alignment of Science and Mathematics Standards and Assessments in Four States*. Washington, DC: Council of Chief State School Officers.

> In his report on the alignment of mathematics curriculum, instruction, and assessment, Webb identifies and describes four levels of depth of knowledge: recall, basic application, strategic thinking, and extended thinking. He describes the learning activities and responses that characterize these levels of depth of knowledge in math and science. The cognitive demand of the NCTM standards are written at the second and third levels, and therefore instruction and assessments should be primarily at the application and strategic thinking levels so that we have aligned our practices with the intent of the standards.

Wolfe, Patricia. 2001. *Brain Matters: Translating Research into Classroom Practice*. Alexandria, VA: Association for Supervision and Curriculum Development.

> Wolfe's book is an easy-to-read guide about how the brain works in relation to learning and teaching. She compiles research from many studies and explains how the findings translate into classroom practice. The chapter devoted to visual thinking is very informative for any teacher of any subject. She writes, "Not only are visuals powerful retention aids, but they also serve to increase understanding. … The ability to transform thoughts into images is often viewed as a test of understanding. But some people appear to process information the other way around, literally seeming to comprehend information by visualizing it. One such person was Albert Einstein, who appeared to process information primarily in images, rather than in written words or spoken language. … One of the defining characteristics of this type of reasoning is the ability to transform abstract concepts into visual images."

Wood, T. and P. Sellers. 1997. "Deepening the Analysis: Longitudinal Assessment of a Problem-Centered Mathematics Program." *Journal of Research in Mathematics Education*, 28 (2): 163–186.

Longitudinal analyses of the mathematical achievement and beliefs of three groups of elementary pupils are presented. The groups consist of those students who had received two years of problem-centered mathematics instruction, those who had received one year, and those who had received textbook instruction. Comparisons are made for the groups using a standardized norm-referenced achievement test from first through fourth grade. Next, student comparisons are made using instruments developed to measure conceptual understanding of arithmetic and beliefs and motivation for learning mathematics. The results of the analyses indicate that after two years in problem-centered classes, students have significantly higher results on standardized achievement measures, better conceptual understanding, and more task-oriented beliefs for learning mathematics than do students who received textbook instruction. In addition, these differences remain after problem-centered students return to classes using textbook instruction. Comparisons of pupils in problem-centered classes for only one year reveal that after returning to textbook instruction, these students' mathematical achievement and beliefs are more similar to the textbook group. Also included are exploratory analyses of the pedagogical beliefs held by teachers before and after teaching in problem-centered classes, and those held by teachers in textbook-centered classes. The results of the student and teacher analyses are interpreted in light of research on children's construction of nonstandard algorithms and the nature of classroom environments.

Zemelman, Steve, Harvey Daniels and Arthur Hyde. 1998. *Best Practice: New Standards for Teaching and Learning In America's Schools,* 2nd ed. Portsmouth, NH: Heinemann.

This book summarizes standards of state-of-the-art teaching and practical descriptions of instructional excellence in six content areas. The authors dispel misconceptions and challenge time honored mathematics practices like memorizing facts; computing pages of sums, differences, products, and quotients; and memorizing rules and procedures for step-by-step proofs. They define the qualities of best practice and encourage teachers and principals to use manipulatives and models, encourage cooperative group work, have students write and talk about mathematics content, and teach using a problem solving approach. Best practice also involves moving away from teaching by telling, stressing memorization without understanding, teaching computation out of context, and testing for grades only.